

Schutz 8.7

$$(a) \quad \bar{h}^\alpha_\alpha = \bar{h}^{\alpha\beta} \eta_{\alpha\beta}$$

$$= \eta_{\alpha\beta} \left(h^{\alpha\beta} - \frac{1}{2} \eta^{\alpha\beta} h \right) \quad \text{by def'n}$$

$$= h^\alpha_\alpha - \frac{1}{2} \eta^{\alpha\beta} \eta_{\alpha\beta} h \quad \eta^{\alpha\beta} \eta_{\alpha\beta} = \text{Tr} \left[\begin{pmatrix} -1 & & \\ & \dots & \\ & & 1 \end{pmatrix} \begin{pmatrix} -1 & & \\ & \dots & \\ & & 1 \end{pmatrix} \right]$$

$$= 4$$

$$= h - 2h = \boxed{-h}$$

$$(b) \quad \bar{h}^{\alpha\beta} = h^{\alpha\beta} - \frac{1}{2} \eta^{\alpha\beta} h$$

$$\Rightarrow h^{\alpha\beta} = \bar{h}^{\alpha\beta} + \frac{1}{2} \eta^{\alpha\beta} h$$

$$= \boxed{\bar{h}^{\alpha\beta} - \frac{1}{2} \eta^{\alpha\beta} \bar{h}}$$

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